are most favorably known among mathematicians were founded by Teixeira. The older of these, entitled *Jornal de sciencias mathematicas e astronomicas*, was founded in 1877. It was superceded in 1905 by the *Annals Scientificos da Academia Polytechnica do Porto*, which is not restricted to mathematics, but has thus far devoted considerable space to this subject. The second number of the current volume begins with an article by P. Appell on the deduction of the polynomials of Hermite from those of Legendre.

From what precedes it is evident that the youngest sister republic cannot be classed with the foremost mathematical countries of the world, but it is equally true that, if we consider her size and population, she has made a very respectable record and is doing so at the present time. It is to be hoped that the new form of government will tend to elevate the educational opportunities of the masses and to put new life also into the higher institutions. In the sixteenth century Lisbon was one of the intellectual centers of Europe, and the later scientific achievements under adverse conditions inspire the hope that with the improvement of these conditions there may come a return of intellectual eminence. Even at the present time some of the Portuguese literature has decided value, both for the investigator and also for those who seek general mathematical knowledge.

ON THE NEW COURSE IN MATHEMATICS IN THE JAPANESE NORMAL SCHOOLS.

By YOSHIO MIKAMI, Phara in Kazusa, Japan.

The Department of Education of the Japanese government recently issued a new course in mathematics for the normal schools; that is, for the schools where the teachers of the primary schools are educated. The courses for men students are not the same as those for women. We begin by giving a brief account of the former.

The course consists of two parts, requiring one and four years respectively. In the former of these (the preliminary course) six hours per week are devoted to mathematics, while the number of hours devoted to this subject during each week of the remaining four years are respectively four, three, three, and two. Hence the total number of hours devoted to mathematics in these normal schools is somewhat less than that of the middle schools.

The six hours of mathematics in the preliminary course are devoted to arithmetic, such as is taught in higher primary schools.*

*In Japan primary schools have two courses, ordinary and higher. The ordinary course lasts six years and the other two years. Children are admitted to the primary school when they reach their sixth year.

In the main course, considerable importance is attached to the practical side of mathematical instruction, emphasizing the connection between various branches of it as much as possible. In this respect the plan differs widely from the prevailing course for the middle school mathematics, where no connection between even arithmetic and algebra is attempted. The new plan is certainly a step towards improvement.

In the first year arithmetic and algebra are united under a common heading, and the subjects taught are integral numbers, the four fundamental operations, decimals, common fractions, negative numbers, integral expressions, linear equations. Geometry is also taught, its theme subjects consisting of the angle, parallels, triangles, and paralellograms, including the areas of rectilinear figures, and the circle. With geometry arithmetical calculations are associated.

In the second year arithmetic, algebra and geometry are all taught under a common heading. The subjects are fractional expressions and fractional equations, square and cube roots, quadratic equations, irrational equations, proportion, similar figures and areas.

In the third year the trigonometric functions of acute angles, together with the solution of triangles, arithmetic and geometric progressions and problems in interest are taught. Besides these, lessons on book-keeping and on the teaching of arithmetic for primary schools are given during the year.

In the fourth year algebra disappears and solid geometry alone is taught. Planes, solid angles, prisms, pyramids, circular cylinders, circular cones, spheres, and the volumes of these solids, are the subjects now considered. Arithmetical mensuration is particularly recommended to be associated with the subjects of solid geometry.

For the women students the preliminary course and the first and third years of the main course are provided each with one hour less than for the men. But all the subjects for the latter are preserved for the women. The only difference is that geometry is begun in the second year, the trigonometric functions are omitted, and that some parts are arranged differently. It will, however, be understood that the subject matter is considerably simpler than that for the men, since the women receive a smaller number of hours of instruction.

Besides the main course of four years, there are established simpler courses of two years and of one year. In the first year of the two years course mathematics is taught four hours a week, and in the second year three hours. Here the subject matter of the first and second years of the four year course is taught in the first year, and that of the third and fourth years in the second year. But it is evident that some parts are necessarily simplified, owing to the shorter time.

In the one year course all the main subjects of the four years of the main course are taught, only with simplifications and omissions.

In addition to the subjects mentioned above, mental arithmetic, the abacus arithmetic with the use of the *soroban*, and the geometrical loci and problems of construction are taught at convenient times. The *soroban* is an abacus that was introduced from China. Its introduction is believed by some writers to have been about 1600, but it may have been made at a far earlier period. However, it was since the middle of the seventeenth century that it was popularized in Japan. Although there was also another sort of abacus, the *sangi*, or calculating pieces, the sole help in daily use for calculations was the *soroban*; for the *sangi* were too cumbersome and better adapted to more complicated calculations. Even since the introduction of the Occidental style of calculation in the middle part of the last century, the *soroban* has not entirely disappeared, and it is still widely used. This is the reason why the *soroban* arithmetic is taught at present in primary schools, together with the Occidental arithmetic.

The adoption of the practical side of mathematical teaching in normal schools will certainly be against the wishes of those who insist that beginners should have theoretical instruction only. But this plan appears to prove successful in training young minds to the assimilation of mathematical ideas; it is especially in agreement with the development of the Japanese character which always looks towards the practical.

It is understood that the new course is not free from faults, but it is recommended to the teacher as a standard, and it is believed that the skill and the power of adaptation already displayed by the Japanese in so many directions will enable them to improve upon it and to develop eventually a still more complete and satisfactory plan. It is worthy of note that Japan is engaged in this development at the same time that all the peoples in the civilized world seem to be considering the possible improvements in the teaching of mathematics.

ON A MEAN DIFFERENCE PROBLEM THAT OCCURS IN STATISTICS.

By H. L. RIETZ, University of Illinois.

1. *Introduction.* In making a comparison of the differences between the highest and lowest examination marks of a pupil in a given subject, with the corresponding difference for the same pupil in some other subjects, E. G. Dexter dealt with data such that, in one subject, say in mathematics, each pupil had ten distinct marks, while in another subject, say in Latin, each pupil had only two or three such marks. In this case, it seems reasonable to expect that, other things being equal, the extreme marks in mathematics would tend to differ more that the extreme marks in Latin.

In considering the question of comparing the average values of the differences between these extreme marks, for a large number of pupils, Dexter proposed to me the following problem:

An urn contains 101 counters marked, 0, 1, 2, ..., 100. Drawings are made at random taking r at a time, always replacing the counters before drawing again. Find the mean difference between numbers on the counters taken two at a time, and the mean difference between the most extreme numbers on counters taken 3, 4, 5, ..., r at a time.

I present the solution here not merely because it seems to be a special problem of some interest, but mainly because of its relation to a general problem in statistics, known as Galton's Difference Problem.*

It may be worth remarking here that the solution of Galton's Difference Problem offers, as a special result, an answer to the question of the most suitable ratio between the values of first and second prizes in a competition.

2. To solve the problem proposed by Dexter, let us define the mean difference as the sum of the products of each separate difference by the probability of its occurrence.

a) The case of drawing two counters at a time.

We may form a difference of 100 in one way, of 99 in two ways, of 98 in three ways, and so on. Hence the mean value is

$$M=\frac{100.1+99.2+\ldots+1.100}{1+2+..,+100}=\frac{\sum_{1}^{100} x(101-x)}{\sum_{1}^{100} x}=34.$$

b) The case of drawing three counters at a time.

We may form a difference of 100 between extremes in 1×99 ways, of 99 in 2×98 ways, of 98 in 3×97 ways, and so on. Hence, the mean difference between extremes is

$$M=\frac{100.99.1+99.98.2+\ldots+2.1.99}{99.1+98.2+\ldots+1.99}=\frac{\sum_{1}^{99} x(101-x)\,(100-x)}{\sum_{1}^{99} x(100-x)}=51.$$

c) The case of drawing r counters at a time.

We may form a difference of 100 in $1\times {}_{99}C_{r-2}$ ways, of 99 in $2\times {}_{98}C_{r-2}$, of 98 in $3\times {}_{97}C_{r-2}$ ways, and so on. From this, we find, by a very slight reduction, that the mean difference between extreme numbers on the r counters is given by

*Pearson, *Biometrika*, Vol. I, pp. 390-399.

$$M = \frac{\sum_{1}^{102-r} (101-x)x(100-x)(99-x)\dots(103-r-x)}{\sum_{1}^{102-r} x(100-x)(99-x)\dots(103-r-x)}$$

To evaluate this expression for any special value of r requires only the sum of series that are integral powers of the natural numbers; that is, of the series $1^s+2^s+3^s+\dots+n^s$.

If, corresponding to each mark from 0 to 100, there are a fixed number t of counters in the urn instead of one only, the problem is solved by a slight extension of the above; since, when $t \geqq r$, there are a certain number of ways in which r counters with equal numbers may be drawn.

On the side of statistical applications, obviously data arranged in frequency groups with respect to some character rarely even approximate to constant values at equal intervals as suggested by the counters in the above problem. For example, examination marks on a percentage basis are, in general, much more frequent at some intervals of the range from 0 to 100 than at others. Thus, as an illustration, the examination marks of a certain group of 1255 students in foreign languages, may be exhibited in frequency groups as follows:

Intervals:	50-52.5	52.5-57.5	57.5-62.5	62.5-67.5	67.5-72.5
Frequency:	1	1	2	6	20

72.5-77.5	77.5-82.5	82.5-87.5	87.5-92.5	92.5-97.5	97.5-
89	195	283	323	307	28

This arrangement of a totality with respect to some character is called a *frequency distribution.* It is a reasonable assumption that the frequency of occurrence follows some law of probability; and, to make the matter fairly simple, we might perhaps assume an ideal system of grading such that the frequency of marks from 0 to 100 should be proportional to the 101 terms in the expansion of $(p+q)^{100}$ where $p+q=1$. But the problem of the mean difference between extreme numbers in drawing numbered counters with such a law of distribution becomes very complicated. On this account, we seek an analytic treatment that will bring in functions to which we can apply the calculus in performing summations.

3. *Continuous treatment of mean differences.* The problem proposed by Dexter has a continuous analogue in the following:

On a line segment AB of length l, r points $x_1, x_2, \dots, x_r$, are selected at random. Find the mean value of $|x_r-x_1|$ where x_1 and x_r denote the extremes of the r points.

For the simple case of two points, this problem is solved in some standard books* on the theory of probability.

It is assumed for this problem that the probability of a point selected belonging to a constant interval ∂x of the line AB is the same no matter where the interval is chosen.

Let x' be the abscissa of any point on l and x that of any point, where $x>x'$. Then the probability of a point first selected belonging to an interval $\partial x'$, the second selected to ∂x, and the remaining $r-2$ to the interval between x' and x is

$$\frac{\partial x'}{l}\cdot\frac{\partial x}{l}\cdot\left(\frac{x-x'}{l}\right)^{r-2}.$$

But there are $r(r-1)$ ways to select a group of 1, 1 and $r-2$ out of r things. Hence, we have for the mean difference

$$r(r-1)\int_0^l\int_0^x \frac{\partial x'}{l}\cdot\frac{\partial x}{l}\left(\frac{x-x'}{l}\right)^{r-2}(x-x')=\frac{r-1}{r+1}l.$$

This result gives, for a constant and continuous distribution from 0 to 100, a mean difference of $33\frac{1}{3}$, when selected in sets of two to compare with 34 for a distribution of a single individual at intervals of one unit, and a mean difference of 50 to compare with 51 when selections are made three at a time.

4. *Galton's Difference Problem.* Let us consider a class of objects and let some numerical mark be attached to the individuals of this class. For example, we may think of statistical data representing the statures of men of a certain class, or the examination marks of a group of pupils. All that is essential is a totality each individual of which is marked by some number. Let the marks be represented as abscissas of points on the x-axis, and let us assume that a function $y=f(x)$ exists such that $y\partial x$ gives the probability that a point, located by the mark of an individual taken at random from the totality, belongs to the interval from x to $x+\partial x$. If a sample of n be selected at random out of the totality, we set the problem of finding the mean difference between the pth and $(p+1)$th individuals of the sample, when the n individuals of the sample are arranged in order of magnitude with respect to the character.

This is Galton's Difference Problem, and we need only add such differences from $p=1$ to $p=n-1$ to solve the problem of the mean difference between extremes in samples of n.

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*Czuber, *Wahrscheinlichkeitsrechnung*, 1903, p. 76.
Borel, *Eléments de la Théorie des Probabilités*, p. 97.

Pearson stated Galton's Problem in its general form, and obtained* for the mean difference between the pth and $(p+1)$th individual

$$D_p=\frac{n!}{(n-p)!\,p!}\int_{-\infty}^{+\infty}\alpha^{n-p}(1-\alpha)^p d\alpha \ldots (1),$$

where

$$\alpha=\int_{-\infty}^{x} f(x)dx \ldots (2).$$

The problem is thus reduced to one of quadrature. Suppose $f(x)$ is the probability function of Gauss, written in the form $f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-x^2/2\sigma^2}$; then, tables of values of α with the argument x are easily accessible. In this case, by quadration, from (1), using Simpson's two-thirds rule and intervals of $0.2\,\sigma$, I have evaluated D_p for $n=2$ and $n=10$. The values for $n=3$ are given by Pearson. The results are:

For $n=2$, $D_1=1.13\,\sigma$,†

$n=3$, $D_1=0.846\,\sigma$, $D_2=0.846\,\sigma$,

$n=10$, $D_1=0.540\,\sigma$, $D_2=0.346\,\sigma$, $D_3=0.282\,\sigma$, $D_4=0.253$,
$D_5=0.247$, $D_6=0.253$, $D_7=0.282\,\sigma$, $D_8=0.346\,\sigma$, $D_9=0.540\,\sigma$.

Let E_n=the mean difference of extreme values when samples of n are taken, then

$$\begin{aligned} E_2&=1.13\,\sigma,\\ E_3&=1.69\,\sigma, \ldots (3).\\ E_{10}&=3.09\,\sigma. \end{aligned}$$

To indicate the significance of these results, we may say that in selecting at random a sample of 10 from a Gaussian distribution, we have a mean difference between extremes of $\frac{3.09}{1.13}=2.73$ times as much as when we take only two at a time, and $\frac{3.08}{1.69}=1.82$ times as much as when we take three at a time.

To apply these results to a reasonable distribution for examination marks, suppose that marks range from 50 to 100, and that the frequencies of occurrence of given marks 50, 51, 52, ... are proportional to the terms in

*Loc. cit., p. 392.

†The last figure in these calculations may be of doubtful value.

the expansion of $(\frac{1}{2}+\frac{1}{2})^{50}$. Then the distribution is well described by a Gaussian function in which

$$\sigma=\sqrt{(50\times\tfrac{1}{2}\times\tfrac{1}{2})}=\frac{5\sqrt{2}}{2}.$$

In this case,

$$E_2=3.99,$$
$$E_3=5.97,$$
$$E_{10}=10.92.$$

The results (3) give a precise notion as to the values of the mean difference between extreme individuals of a small sample taken at random from a totality that is distributed in accord with Gauss's law.

Since many frequency distributions not well described by Gauss's law are well described by its generalizations, it seems likely that results, such as concern us here, when derived from Gauss's curve apply at least roughly to many more general types of frequency distributions that occur in statistics. To illustrate, the distribution of examination marks given on p. 237 is not at all well fitted by numbers proportional to the terms in the expansion of $(\frac{1}{2}+\frac{1}{2})^{50}$; but, if we use the ungraduated values there given to evaluate (1) by quadrature, we obtain, for the mean difference in taking sets of two, the value 7.71. But, if we compute σ as the square root of the mean square of the deviations of observations from their mean value, the result is 7.42. Then, from (3), $E_2=8.38$ to compare with 7.71; and, hence, the result from the assumption of the law of Gauss gives at least a good general notion of the mean difference in question.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

342. Proposed by E. B. ESCOTT, Ann Arbor, Michigan.

Prove that $\frac{1}{1.2.3.4}+\frac{1}{5.6.7.8}+\ldots=\frac{1}{4}\log 2-\frac{1}{24}\pi$. [Hobson's *Plane Trigonometry*, page 348.]

Solution by V. M. SPUNAR, Cleveland, Ohio, and the PROPOSER.

The general term is

$$\frac{1}{(4n-3)(4n-2)(4n-1)4n} = \tfrac{1}{6}\left(\frac{1}{4n-3} - \frac{3}{4n-2} + \frac{3}{4n-1} - \frac{1}{4n}\right)$$

$$= \tfrac{1}{6}\left(\frac{2}{4n-3} - \frac{2}{4n-2} + \frac{2}{4n-1} - \frac{2}{4n}\right) - \tfrac{1}{6}\left(\frac{1}{4n-3} - \frac{1}{4n-1}\right) - \tfrac{1}{6}\left(\frac{1}{4n-2} - \frac{1}{4n}\right).$$

Therefore the series may be written

$$\tfrac{1}{3}(1-\tfrac{1}{2}+\tfrac{1}{3}-\tfrac{1}{4}+\ldots) - \tfrac{1}{6}(1-\tfrac{1}{3}+\tfrac{1}{5}-\tfrac{1}{7}+\ldots) - \tfrac{1}{12}(1-\tfrac{1}{2}+\tfrac{1}{3}-\tfrac{1}{4}+\ldots)$$

$$= \tfrac{1}{4}\log 2 - \tfrac{1}{6}\tan^{-1}1 = \tfrac{1}{4}\log 2 - \tfrac{1}{24}\pi.$$

343. Proposed by THEODORE L. DeLAND, Treasury Department, Washington, D. C.

A, on contracting to execute a piece of work for $300 and finding after working alone one day that he had finished but 1% of the entire work, engaged B to assist him at the beginning of the second day, with the understanding, that B on each day was to do 6% as much work as had been completed previously, while A each day was to do an amount of work equal to 1% of the unfinished work at the close of the day before. At the completion of all the work the $300 were divided between A and B in proportion to the amount of the work each had performed.

Required—(1) The number of days to do the work; (2) on which day would the daily earnings of A and B be the same; and (3) the amount of money each was paid under the agreement.

Solution by the PROPOSER.

Let $r=1\%$ and $r_1=6\%$; and let x, a variable, =the time in days to do the whole work, or 1; and let the whole work completed to the end of the days 0, 1, 2, 3, ..., x, $x+1$, ..., be represented by the functions u_0, u_1, u_2, ..., u_x, u_{x+1}, ... The whole work completed to the end of the day $x+1$, or u_{x+1}, is equal to the work completed to the end of x days, or u_x, plus the part completed by A on the day x, or $r(1-u_x)$, plus the part completed by B on the day x, or $r_1 u_x$. Equate the functions and have:

$$u_{x+1} = u_x + r(1-u_x) + r_1 u_x \ldots (1);$$

$$\text{or, } u_{x+1} - (1-r+r_1)u_x = r \ldots (2).$$

Give the equation numerical values and we have

$$u_{x+1} - (1.05)u_x = 0.01 \ldots (3).$$

Equation (3) belongs to the Calculus of Finite Differences. Integrate it and have

$$u_x - C(1.05)^x = -0.2 \ldots (4).$$

Equation (4) is true for all values of x, and is therefore true when $x=0$. When $x=0$, $C=0.2$; and this value of C gives

$$u_x = 0.2[(1.05)^x - 1] \ldots (5).$$

To find (1) the time to complete the work. When the work is completed $u_x=1$. Substitute this value of u_x in (5) and have, $(1.05)^x=6$; or $x\log(1.05)=\log 6$; or $x=\log 6/\log(1.05)=36.72+$days.

To find (2), when the earnings of each are the same, in equation (1) transfer u_x to the first member and have

$$u_{x+1} - u_x = r(1-u_x) + r_1 u_x \ldots (6).$$

As u_{x+1} is the whole work completed in $x+1$ days, and u_x is the whole work completed in x days, their difference is the work completed on the day $x+1$. Substitute in the second member of (6), the value of u_x from (5) and have

$$u_{x+1} - u_x = 0.002[6 - (1.05)^x] + 0.012[(1.05)^x - 1] \ldots (7).$$

In (7), for $x+1$ write x, as x is a variable, and have

$$u_x - u_{x-1} = 0.002[6 - (1.05)^{x-1}] + 0.012[(1.05)^{x-1} - 1] \ldots (8).$$

The first member of (8) is the work completed in x days, and the two terms of the second member show the work completed by A and B, respectively, on the day x, and as these two terms, under the conditions of the problem, must be equal, equate them, reduce, and have

$$6 - (1.05)^{x-1} = 6[(1.05)^{x-1} - 1] \ldots (9);$$

or $7(1.05)^{x-1}=12$; or $(1.05)^{x-1}=12\div 7$; or $(x-1)\log(1.05)=\log(12\div 7)$; or $x=1+\log(12\div 7)/\log(1.05)=12.05$ days, or 12 days.

To divide the money (3) recur to equation (8) and observe that the exponents of the two terms in the second member are one degree lower than

the subscript x in u_x. This is a general law and it enables us to generate the 36 equations of work completed as x takes different values from 1 to 36.

End of 1 day, $u_1-u_0=0.002[6-(1.05)^1]+[0]\ldots(10)$;

End or 2 days, $u_2-u_1=0.202[6-(1.05)^1]+0.012[(1.05)^1-1]\ldots(11)$;

. . . . . .

End of 35 days, $u_{35}-u_{34}=0.002[6-(1.05)^{34}]+0.012[(1.05)^{34}-1]\ldots(44)$; and

End of 36 days, $u_{36}-u_{35}=0.002[6-(1.05)^{35}]+0.012[(1.05)^{35}-1]\ldots(45)$.

Add the equations and have

$$u_{36}-u_0=0.002\{216-[(1.05)^0+(1.05)^1+\ldots+(1.05)^{35}]\}$$
$$+0.012[(1.05)^1+(1.05)^2+\ldots(1.05)^{35}-35]\ldots(46).$$

Sum the first term of the second member of (46) for the work completed by A in 36 days, and sum the second term for the work completed by B in 35 days, and have:

$$\text{for A's work, } 0.002\{216-20[(1.05)^{36}-1]\}=0.2403+;$$
$$\text{and for B's work, } 0.012\{21[(1.05)^{35}-1]-35\}=0.7180+.$$

A's work for 36 days+B's work for 35 days=0.2403+0.7180=0.9583+.

The unfinished work=1−0.9583=0.0417−; work to be finished by A and B in 0.72 day.

For A's unfinished part we have (0.0417)(0.01)(0.72)=0.0003; and 0.2403+0.0003=0.2406=the total part completed by A.

For B's unfinished part we have (0.8583)(0.06)(0.72)=0.0414; and 0.7180+0.0414=0.7594, the total part completed by B.

A's total+B's total=0.2406+0.7594=1, as it should.

A's share of the money, therefore, =\$300×(0.2406)=\$72.18; and B's share=\$300×(0.7594)=\$227.82.

Also solved by V. M. Spunar.

GEOMETRY.

369. Proposed by W. J. GREENSTREET, A. M., Editor, Mathematical Gazette, Stroud, England.

Prove by inversion that if two circles cut at a given angle, touch each a given circle, and pass each through the same fixed point, then shall the envelope of the points of contact be a conic.

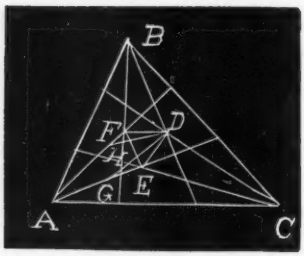
No satisfactory solution of this problem has been received.

370. Proposed by R. C. ARCHIBALD, Paris, France.

The trisectors of the angles of any triangle ABC are, in order, AF, AE, CE, CD, BD, BF. Show synthetically that D, E, F are the vertices of an equilateral triangle.

Solution by A. H. HOLMES, Bruntwick, Me.

Let ABC be the triangle with AE, AF trisectors of $\angle BAC$; BF, BD trisectors of $\angle ABC$, and CE, CD trisectors of $\angle ACB$. Draw DE, DF, and EF. Then DEF is an equilateral triangle. From D draw DG parallel to AF and cutting AE in G. Take, on AF, $AH=DG$. Then $\angle GDH = \angle EAF = \frac{1}{3}\angle BAC$.



Suppose the triangle GED to be moved so that GE is colinear with AF and the point E is at the point F. Then since DG is parallel to AF and $\angle EAF = \angle FAB$, DG will be parallel to AB.

$\therefore \triangle DEG$ and $\triangle AFB$ are similar, and $\angle EDG = \angle ABF = \frac{1}{3}\angle ABC$. Similarly, it may be shown that $\angle FDH$ is equal to $\frac{1}{3}\angle ACB$.

$\therefore \angle EDF = \frac{1}{3}(\angle ABC + \angle ACD + \angle BAC) = 60^\circ$. In the same way, $\angle DEF$ is shown to be equal to 60°.

$\therefore \triangle DEF$ is an equilateral triangle.

371. Proposed by W. S. HUGHES, Student, Williams College.

A right circular cone is cut by two parallel planes, one passing through the vertex, and each cutting both nappes. Are the straight lines which constitute the first section parallel to the asymptotes of the hyperbola forming the other section?

Solution by FRANK LOXLEY GRIFFIN, Ph. D., Assistant Professor of Mathematics, Williams College.

As the X-axis take the axis of the cone, and as the Y-axis the intersection of the first cutting plane with the plane through the vertex perpendicular to the X-axis. Then the equation of the cone is $y^2+z^2=m^2x^2$, where m denotes the tangent of one-half the vertex angle of the cone. In the plane XOZ rotate the axes OX and OZ through an angle α, such that OX shall be in the first cutting plane. Then the cone and the cutting planes are given respectively by

$$(1) \qquad y^2+(x'\sin\alpha+z'\cos\alpha)^2=m^2(x'\cos\alpha-z'\sin\alpha),$$

$$(2),\ (3) \qquad z'=0, \qquad z'=d,$$

where d is the distance between the cutting planes. Now the straight lines of the first section are given by (2) and the equation obtained by making $z'=0$ in (1), say $B^2x'^2-y^2=0$. And the hyperbola is given by (3) and an

equation obtained from (1) for $z'=d$, whose second degree terms, being those of (1) not containing z', are the same: $B^2(x'-k)^2-y^2=l^2$. Thus the asymptotes $y=\pm B(x'-k)$, $z'=d$ are parallel to the first lines $y=\pm Bx'$, $z'=0$.

Also solved by S. G. Barton and V. M. Spunar.

MECHANICS.

249. Proposed by the late G. B. M. ZERR, Ph. D.

A load P is supported by three strings of equal size attached at the vertices of a triangle, sides a, b, c lying in a horizontal plane. The load is vertically under the centroid of the triangle at a distance h from it. Find the stresses in the strings.

Solution by the PROPOSER.

Let h_1, h_2, h_3 be the medians of the triangle; T_1, T_2, T_3 the stresses on the strings attached to A, B, C, respectively; D, the point where the load is fixed; G, the centroid; H, the mid-point of BC.

$\angle ADG=\theta$, $\angle BDG=\phi$, $\angle CDG=\psi$, $\angle BGH=\rho$, $\angle CGH=\mu$.

$$p=\cos\theta=3h/\sqrt{(9h^2+4h_1^2)}=3h/\sqrt{(9h^2+2b^2+2c^2-a^2)}.$$

$$q=\cos\phi=3h/\sqrt{(9h^2+2a^2+2c^2-b^2)}.$$

$$r=\cos\psi=3h/\sqrt{(9h^2+2a^2+2b^2-c^2)}.$$

$$m=\sin\phi\sin\rho=3ab\sin C/\sqrt{[(9h^2+2a^2+2c^2-b^2)(2b^2+2c^2-a^2)]}.$$

$$n=\sin\psi\sin\mu=3ab\sin C/\sqrt{[9h^2+2a^2+2b^2-c^2)(2b^2+2c^2-a^2)]}.$$

Let E=Young's modulus, β=sectional area of string, p_1=elongation AD, p_2=elongation BD, p_3=elongation CD. Then

$$P=pT_1+qT_2+rT_3\ldots(1),$$
$$mT_2=nT_3\ldots(2),$$
$$T_1p_1+T_2p_2+T_3p_3=\text{minimum}.$$

Now $p_1=\dfrac{AD.T_1}{E\beta}$, $p_2=\dfrac{BD.T_2}{E\beta}$, $p_3=\dfrac{CD.T_3}{E\beta}$.

$AD.p=BD.q=CD.r=h$.

Hence, $\dfrac{h}{E\beta}\left(\dfrac{T_1^2}{p}+\dfrac{T_2^2}{q}+\dfrac{T_3^2}{r}\right)=\text{minimum}\ldots(3)$.

From (1), (2) and (3) we get

$$pdT_1+qdT_2+rdT_3=0\ldots(4),$$
$$mdT_2=ndT_3\ldots(5),$$
$$T_1dT_1/p+T_2dT_2/q+T_3dT_3/r=0\ldots(6).$$

Eliminating dT_1, dT_2, dT_3 between (4), (5) and (6), we get

$$T_1(qn+rm)qr=T_2p^2m+T_3p^2qm\ldots(7).$$

From (1), (2) and (7) we get

$$T_1=\frac{Pp^2(rn^2+qm^2)}{p^3(rn^2+qm^2)+qr\,(qn+rm)}.$$

$$T_2=\frac{Pqrn(qn+rm)}{p^3(rn^2+qm^2)+qr\,(qn+rm)}.$$

$$T_3=\frac{Pqrm(qn+rm)}{p^3(rn^2+qm^2)+qr\,(qn+rm)}.$$

ERRATA.—Begin with problem 240. Mechanics, page 194, Vol. XVI, and number them consecutively through Vol. XVII. Problem 247 in the last issue should be 248. Note that 244 on page 48, Vol. XVII is the same as 241, page 21.

250. Proposed by C. N. SCHMALL, New York City.

A smooth circular table is surrounded by a smooth vertical rim. A ball of elasticity e is projected from a point at the rim in a line making an angle ϕ with the radius through that point. Show that the ball will return to the starting point after the second impact if

$$\tan\phi=\sqrt{\frac{e^3}{e^2+e+1}}.$$

Solution by the late G. B. M. ZERR, Ph. D.

Let A be the point of projection; B, C the points of first and second impact; O, the center of the table; $\angle OAB=\angle OBA=\phi$, $\angle OBC=\angle OCB=\theta$, $\angle OCA=\angle OAC=\psi$.

Then $\tan\phi=e\tan\theta$, $\tan\theta=e\tan\psi$.

Hence, $\tan\phi=e\tan\theta=e^2\tan\psi$. Also, $\phi+\theta+\psi=\frac{1}{2}\pi$.

$\therefore 1=\tan\phi\tan\theta+\tan\phi\tan\psi+\tan\theta\tan\psi$. Whence

$$1=\tan^2\phi\left(\frac{1}{e}+\frac{1}{e^2}+\frac{1}{e^3}\right).$$

Hence, $e^3=(1+e+e^2)\tan^2\phi$, from which we obtain,

$$\tan\phi=\sqrt{\frac{e^3}{e^2+e+1}}.$$

Solved in a similar manner by J. Scheffer.

PROBLEMS FOR SOLUTION.

GEOMETRY.

379. Proposed by G. I. HOPKINS, Manchester, N. H.

Construct the triangle, having given base, vertical angle, and ratio of its altitude to difference of other two sides.

380. Proposed by W. J. GREENSTREET, A. M., Stroud, England.

$A\ B\ C\ D$ is a quadrilateral, sides in order a, b, c, d, and $B+D=\theta$. Express the diagonals in terms of a, b, c, d, θ.

MECHANICS.

355. Proposed by the late G. B. M. ZERR, Ph. D.

Assuming the resilience of volume of mercury to be constant at all depths and to be 54.20×10^{10} in *C. G. S.* units and that a mile = 160933 centimeters. Find the depth of an ocean of mercury at a point where its density is double the surface density, 13.596.

356. Proposed by the late G. B. M. ZERR, Ph. D.

A cantilever beam length a is loaded with c pounds per running foot at its fixed end and increases uniformly to b pounds per running foot at its free end. Find the deflection at the free end due to this load.

357. Proposed by W. J. GREENSTREET, M. A., Stroud, England.

A portion of a circular cylinder cut off by two planes through the axis rests with its curved surface on two rough horizontal rails parallel to its axis, the coefficients of friction μ_1, μ_2 at upper and lower rails respectively. If the body is in limiting equilibrium at both rails when the plane through the axis and the center of gravity is perpendicular to both rails, find the distance of the center of gravity in terms of the distance betwen the rails, the inclination of their plane to the horizon, and the cofficients of friction.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

177. Proposed by J. EDWARD SANDERS, U. S. Weather Bureau Office, Chicago, Ill.

Factor (if possible) 1,111,111,111,111,111,111.

NOTES AND NEWS.

Dr. George Bruce Halsted has been elected Corresponding Member of the Société des Sciences Physiques et Naturelles de Bordeaux, in whose Mémoires appeared his work La contribution non euclidienne à la philosophie. F.

Messrs. Ginn and Company announce the publication of a book entitled *The Hindu-Arabic Numerals*, by David Eugene Smith, of Columbia University, and Louis C. Karpinski, of the University of Michigan. The work gives a complete story of the rise and development of the numerals, and is illustrated with numerous facsimiles from early inscriptions and manuscripts, most of which have not hitherto been published in connection with this subject, and all of which contribute to a very marked degree to an understanding of the problem. S.

The Mathematics Club of Columbia University is a new organization for the purpose of promoting personal and intellectual relations among the graduate students and faculty in the department of mathematics. Regular meetings of an informal character will be held for the reading and discussion of papers. This organization is in addition to the more formal Mathematical Colloquium of long standing in the department. A similar organization has existed at the University of Chicago for several years, to supplement the work of the Mathematical Club and Seminar and to form a rallying point for mathematical students early in their graduate course. S.

In the Central Association of Science and Mathematics Teachers during the past few years, important reports have been presented on the teaching of biology, chemistry, physics, algebra, geometry, and unified mathematics. In addition to these formal reports numerous symposiums and discussions have been published in *School Science and Mathematics*. At the recent annual meeting in Cleveland, there was presented an important report on the fundamental findings of all the previous documents, including both those common to the various subjects and those peculiar to special subjects. This report is well worth the careful study of all teachers of mathematics. It shows the constructive and coherent activity of this association. Copies may be obtained from the press of *School Science and Mathematics*, 2059 East 72nd Place, Chicago, Illinois. S.

BOOKS.

Text Book on Practical Astronomy. By George L. Hosmer, Assistant Professor of Civil Engineering, Massachusetts Institute of Technology. 8 vo. Cloth, vii+205 pages. 78 figures. Price, $2.00 net. New York: John Wiley & Sons.

The purpose of this volume, we are told, is to furnish a text in practical astronomy, especially adapted to the needs of civil engineering students who can devote but little time to the subject and who are not likely to take up the advanced study of astronomy. The text deals, therefore, largely with that class of observations which can be made with surveying instruments, the refinements of observations and the rigorous mathematical formulation of principles being left for the theoretical student to supply.

The work is in full agreement in its methods with the spirit of modern education, and will find a useful place in practical teaching. F.

An Elementary Treaties on the Dynamics of a Particle and of Rigid Bodies. By S. L. Loney, M. A., Professor of Mathematics at the Royal Holloway College (University of London), sometime Fellow of Sidney Sussex College, Cambridge. 8 vo. Cloth, viii+374 pages. Price, $4.00. Cambridge University Press, Cambridge, England. G. P. Putnam's Sons, American Agents.

This treatise develops the theory of the dynamics of a particle of a rigid body, with that lucidity of expression, simplicity and directness of presentation, and logical consistency so characteristic of Professor Loney's Mathematical Texts. The book is intended for both engineering and mathematical students. A fair working knowledge of the Differential and Integral Calculus, as well as of Differential Equations, is required in order to read it, though all the Differential Equations required are solved in an appendix.

A large number of interesting problems, chiefly collected from University and College examinations, are included, and many problems are solved, thus giving the student a good idea how the various problems may be attacked.

The typography and mechanical execution of the work is all that could be desired. F.

Mechanics. By John Cox, M. A., F. R. S. C., Honorary LL. D., Queen's University, Kingston; formerly Professor of Physics in McGill University, Montreal; sometime Fellow of Trinity College, Cambridge, England. 8 vo. Cloth, xiv+332 pages. Price, $2.75. Cambridge: The University Press. G. P. Putnam's Sons, American Agents.

This book may be considered as a protest against the prevailing method of presenting the subject of mechanics, viz.: The method beginning the subject by collection of definitions of mass, motion, force, matter, etc.. comprising the first chapter and the second taking up the mathematical study of motion in the abstract, and so through a course which leaves the student with the notion that he has been studying mathematics pure but not simple. The author has fashioned his course of study somewhat after the famous treatise of Professor Ernst Mach. In writing the book the author has kept in view six aims, of which we mention the following:

(1) First and throughout, to make a text-book of mechanical principles, avoiding as far as possible merely mathematical difficulties; (2), to develop the principles in their historical order; (3), to bring out incidentally the points of philosophic interest and the method of science; and (4), to interest the student in the personality of the great pioneers of the science and if possible have them refer to their writings.

The course as presented in this volume has been tested by the author with his classes in McGill University, and has proved very satisfactory to him.

We believe that it would be well to have some such course as this to serve as an introduction to the study of mechanics, without regard to the future use the student wishes to make of the knowledge thus gained. The author believes that, "until mechanics is clad in its historical flesh and blood, it will remain the dull and tiresome subject that has convinced so many generations of students that an abysmal gulf separates theory from practice."

The text also contains portraits of Archimedes, Galileo, Huyghens, and Newton. F.

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